

12-3 The Quadratic Formula

Objective: To learn the quadratic formula and use it to solve equations.

The Quadratic Formula

The solutions of a quadratic equation in the form of $ax^2 + bx + c = 0$, $a \neq 0$ and $b^2 - 4ac \geq 0$ are given by the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 1 Use the quadratic formula to solve $3x^2 + 5x - 2 = 0$.

Solution $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where $a = 3$, $b = 5$, and $c = -2$.

$$x = \frac{-5 \pm \sqrt{(5)^2 - 4(3)(-2)}}{2(3)} \quad \text{Substitute the given values of } a, b, \text{ and } c.$$

$$= \frac{-5 \pm \sqrt{25 + 24}}{6}$$

$$= \frac{-5 \pm \sqrt{49}}{6} = \frac{-5 \pm 7}{6}$$

$$x = \frac{-5 + 7}{6} = \frac{2}{6} = \frac{1}{3} \quad \text{or} \quad x = \frac{-5 - 7}{6} = \frac{-12}{6} = -2$$

The check is left to you. The solution set is $\{\frac{1}{3}, -2\}$.

Use the quadratic formula to solve each equation.

1. $x^2 + 3x - 10 = 0$ $\{-5, 2\}$
2. $x^2 - 8x + 7 = 0$ $\{7, 1\}$
3. $x^2 + 2x - 3 = 0$ $\{-3, 1\}$
4. $x^2 - 14x + 24 = 0$ $\{2, 12\}$
5. $n^2 + 5n - 6 = 0$ $\{-6, 1\}$
6. $x^2 - 6x - 40 = 0$ $\{-4, 10\}$
7. $2x^2 + 3x - 2 = 0$ $\{-2, \frac{1}{2}\}$
8. $3u^2 - 5u - 2 = 0$ $\{2, -\frac{1}{3}\}$
9. $3x^2 - 10x - 8 = 0$ $\{4, -\frac{2}{3}\}$
10. $3x^2 - 2x - 1 = 0$ $\{-\frac{1}{3}, 1\}$
11. $2x^2 - 5x - 7 = 0$ $\{-1, \frac{7}{2}\}$
12. $5x^2 + 6x - 8 = 0$ $\{-2, \frac{4}{5}\}$

Example 2 Use the quadratic formula to solve $x^2 = x - 6$.

Solution $x^2 - x + 6 = 0$ Rewrite the equation in standard form.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \text{ where } a = 1, b = -1, \text{ and } c = 6.$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(6)}}{2(1)} = \frac{1 \pm \sqrt{1 - 24}}{2} = \frac{1 \pm \sqrt{-23}}{2}$$

Since $\sqrt{b^2 - 4ac} = \sqrt{-23}$ and $\sqrt{-23}$ isn't a real number, there is *no real solution*.

12-3 The Quadratic Formula (continued)

- Use the quadratic formula to solve each equation.
13. $x^2 - 4x + 6 = 0$ **No sol.**
 14. $2x^2 = 3x - 1$ $\{\frac{1}{2}, 1\}$
 15. $x^2 - 4x = 30$
 16. $2x^2 + 2x + 5 = 0$ **No sol.**
 17. $4x^2 + 20x = -9$
 18. $3x^2 - 3x + 4 = 0$ **No sol.**
 17. $\{-\frac{1}{2}, -\frac{9}{2}\}$ $\{2 + \sqrt{34}, 2 - \sqrt{34}\}$

Example 3 Use the quadratic formula to solve $2x^2 - 3x - 4 = 0$. Give irrational roots in simplest radical form and then approximate them to the nearest tenth. You may wish to use a calculator.

Solution $2x^2 - 3x - 4 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \text{ where } a = 2, b = -3, \text{ and } c = -4.$$

$$x = \frac{3 \pm \sqrt{9 - 4(2)(-4)}}{2(2)} \quad \text{Substitute the given values of } a, b, \text{ and } c.$$

$$= \frac{3 \pm \sqrt{9 + 32}}{4} \quad \text{Simplify.}$$

$$= \frac{3 \pm \sqrt{41}}{4}$$

$$\text{Since } \sqrt{41} \approx 6.40, x \approx \frac{3 + 6.4}{4} = 2.35 \approx 2.4$$

$$\text{or } x \approx \frac{3 - 6.4}{4} = -0.85 \approx -0.9$$

The check is left to you.

The solution set is $\{\frac{3 + \sqrt{41}}{4}, \frac{3 - \sqrt{41}}{4}\}$ or $\{2.4, -0.9\}$.

Use the quadratic formula to solve each equation. Give irrational roots in simplest radical form and then approximate them to the nearest tenth. You may wish to use a calculator.

19. $2x^2 = 8x - 5$
20. $3x^2 + 2x = 2$
21. $x^2 - 4x - 10 = 0$
22. $x^2 - 4x - 2 = 0$
23. $2x^2 - 4x + 1 = 0$
24. $3x^2 - 8x + 2 = 0$
25. $2x^2 + 1 = 3x$ $\{\frac{1}{2}, 1\}$
26. $3x^2 + x = 2$ $\{-1, \frac{2}{3}\}$
27. $4x^2 - 11x = 3$ $\{3, -\frac{1}{4}\}$

1. $\{-3, 7, \text{ and the real numbers between } -3 \text{ and } 7\}$
3. $\{\text{the real numbers between } -4 \text{ and } 1\}$
4. $\{3 \text{ and the real numbers between } 0 \text{ and } 3\}$

Mixed Review Exercises

Solve each open sentence and graph its solution set. Graphs given at the back of this Answer Key.

1. $|x - 2| \leq 5$
2. $2|y + 5| = 4$ $\{-3, -7\}$
3. $|2n + 3| < 5$
4. $1 < 2z + 1 \leq 7$
5. $\sqrt{x} = 5$ $\{25\}$
6. $\sqrt{5n + 1} = 6$ $\{7\}$
7. $2\sqrt{2x} = 12$ $\{18\}$
8. $|3 + 2k| = 11$ $\{4, -7\}$
9. $3|2 - m| = 12$ $\{-2, 6\}$

Solve by completing the square.

10. $x^2 - 8x + 12 = 0$ $\{6, 2\}$
11. $3x^2 + 6x = 0$ $\{0, -2\}$
12. $c^2 - c = 12$ $\{-3, 4\}$